EARLY CHILDHOOD ASSESSMENT IN MATHEMATICS MANUAL

NYC Department of Education

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Children First Initiative

Department of Science, Technology, Engineering and Mathematics

CONTENTS

Acknowledgments

This 2009 edition is an update of the Early Childhood Assessment in Mathematics, K-2 that was originally adapted for the New York City Department of Education, with permission from the State of Victoria, Department of Education and Training, in 2006.

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District 7: Myrta Rivera, Superintendent; Maria Rodriquez, Director of Early Childhood; PS 154; Cynthia Ballard, Principal; Peri Wexler, Alexis Garcia, Denise Green, Michelle Cota, Carolyn Rivas, Denise Adme, Adrianne Smith, Jennifer Lyon, Doug Fisher, Addie Miranda, teachers; and Cheryl Hanily; Math Consultant.

District 13: Lester W. Young, Superintendent; Viola Abbott, District Director Math, PS 282; Doreen Corkley, Principal; Magalie Alexis, Assistant Principal; Laryza Martell, Adrienne Moore, Francine Minors; Freddie Mae Jordan, Annette Kennedy, Rosa Perez-Leonetti, teachers; and Mark Waters, Math Consultant.

District 20: Vincent Grippo, Superintendent; Trudy Aducci, District Director Math; PS 314; Catherine Cortes, Principal; Liset Isaac, Assistant Principal; Camille Paolucchi, Kimberley Huang, Maryann Fredella, Elizabeth Natal, Danielle Pietropinto, Denise Accetta, teachers; and Pam Montgomery, Math Consultant.

District 26: Claire McIntee, Superintendent; Iris Feldman, District Math Coordinator; PS 173; Joel Schulman, Principal; Diane Hobbs, Assistant Principal; Theresa Maillard, Ilene Savoy, Anne Shandley, Cheryl Walker, Traci Dolinsky, Linda Bernstein, teachers; and Pam Montgomery, Math Consultant.

1. INTRODUCTION

- *Why assess young children's mathematical understandings?*
- *Assessment before teaching*
- *The mathematics interview*

Why assess young children's mathematical understandings?

Children of similar ages often know different things. For example, some young children starting Kindergarten do not recognize any numerals from 0-10. However, about one third of these children already recognize all these numerals, and some even know many two-digit numerals. Most children beginning Grade 1 count by ones accurately, and know a few skip counting sequences, but some children are still coordinating one-to-one counting of objects.

Instruction is most effective if it matches the child's learning needs. If a child already knows something, and we spend instructional time "teaching" this, this child's understandings will not be extended. Similarly, if we spend instructional time "teaching" something well beyond a child's comprehension, they are likely to become confused. In both cases, the child's opportunity to learn has been wasted.

The way to determine children's learning needs is to assess what each child already understands in mathematics, and to find areas where each child begins to experience difficulties.

Assessment before teaching

Traditionally, assessment has been conducted after teaching. This means that students" understandings are revealed after instruction. The teacher may not be aware of students who have struggled throughout the instructional time, or of students who already understand the concepts being "taught'. By then it is too late to assist these students, as the instruction has moved on to another topic.

Current educational practices advocate assessing students before teaching, and throughout teaching, as well as after

teaching. Assessment before teaching enables a teacher to find out the range of understanding within their class. For example, early in Grade 1, some children will be ready for instruction in "counting on" to add two collections. Other children in the same grade may still need experiences counting all objects from one. Still other children may already be counting on confidently, and so are ready to learn initial addition strategies to find the total mentally. Once a teacher knows this, they can then design appropriate instruction for all these students.

The mathematics interview

Traditionally, pen and paper assessment has been used to determine students" understandings in mathematics. This type of assessment typically shows whether children can give correct answers to mathematical problems, but it does not enable teachers to find out about students" thinking.

The mathematics interview is a rich assessment technique. The teacher asks the child to solve several mathematical tasks, and to explain their thinking as they proceed. The teacher observes and questions the child. This gives the teacher access to how each child figures out particular mathematical problems. Each child's understandings and current strategies can be identified, as well as any misconceptions they may have.

Mathematics interviews train teachers to observe children closely as they work, and to question children about their thinking. These practices can be transferred to the daily mathematics lesson, with teachers closely observing and questioning individual children as they work, and children becoming used to explaining their thinking.

As mathematics interviews are conducted orally and with materials, children do not need to be able to read or write to reveal their understandings. This is of particular benefit in Grades Kindergarten and Grade 1, and with ELL students.

An excellent reference with background information on interviewing as a mathematics assessment technique is *The Teacher's Guide to Flexible Interviewing in the Classroom* by Ginsburg, Jacobs and Lopez (1998, Needham Heights: Allyn & Bacon).

2. RESEARCH BACKGROUND

- *International research*
- *New York City classroom trials*
- *Some findings from the New York City classroom trials*

International research

Young children's learning of mathematics has been increasingly researched and described. Educational writers and researchers such as Gelman and Gallistel (1978), Steffe, von Glaserfeld, Richards & Cobb (1983), Labinowicz (1985), Hughes (1987), Baroody (1987), Fuys & Liebov (1993), Kamii (1989), Carpenter, Fennema, Loef Franke, Levi & Empson (1999), Kamii and Housman (2000), Twomey Fosnot & Dolk (2001) and Clements (2000) have revealed the significant mathematical learning that very young children can achieve.

Particularly in the learning of number and arithmetic, common patterns of development have been identified. For example, very young children learning to count will say "strings" of counting words, which might not follow the conventional sequence: something like "One, two, three, four, five, six, ten, seven, nine," Later, these children confidently use the sequence from 1-9 to count between "decade" numbers, but have trouble "bridging" to the next decade number, for example after correctly counting 41 through 49, they may say "Forty-ten," or "Seventy". Understanding these typical patterns of growth can make teachers more aware of children's mathematical development, and therefore better able to match instruction to children's needs.

This mathematics interview was developed following an Australian research project, the Early Numeracy Research Project. This was conducted in Australia by the Victorian Department of Education, Australian Catholic University and Monash University from 1999 to 2001. Five thousand children in early childhood classrooms were assessed over three years using a structured mathematics interview (Department of Education, Employment and Training, 2001). This interview was matched to a "framework" of research-based early mathematical learning. The framework and interview were designed to be to be practical and useful to early childhood teachers (Clarke, Cheeseman, Clarke, Gervasoni, Gronn, Horne, McDonough, Montgomery, Rowley & Sullivan, 2001).

New York City classroom trials

In 2002, the Division of Instructional Support, New York City Board of Education engaged several mathematics education consultants to develop a similar mathematics interview that would suit Grades K-2 students and teachers in New York City schools. The interview and the assessment items were adapted from the Early Numeracy Research Project (Clarke, et. al., 2001). Mapping to both New York City and New York State mathematics curricula was conducted, with appropriate adjustments being made (Board of Education of the City of New York, 2001; State Education Department).

During the 2002-2003 school year, the assessment tool was trialed in four elementary schools in New York City. The schools were located in the Bronx, Brooklyn, and Queens. Personnel were as follows:

> ***District 7:*** *Myrta Rivera, Superintendent; Maria Rodriquez, Director of Early Childhood; PS 154; Cynthia Ballard, Principal; Peri Wexler, Alexis Garcia, Denise Green, Michelle Cota, Carolyn Rivas, Denise Adme, Adrianne Smith, Jennifer Lyon, Doug Fisher, Addie Miranda, teachers; and Cheryl Hanily; Math Consultant.*
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Six teachers from each school were involved. They attended four workshop days, were coached in administering the interview, interviewed students from their classes at various times throughout the school year, observed demonstration mathematics lessons, co-taught mathematics lessons, and provided feedback about their students" understandings in mathematics. A Math Consultant was assigned to each school. This consultant visited each school between fifteen and seventy days across the school year. The range of trial classrooms included bilingual classes, inclusion classes, heterogeneous classes and homogeneous classes. The Division of Instructional Support, New York City Board of Education, along with educational consultants, made revisions to the assessment tool mid-year following feedback from the classroom teachers.

3. THE EARLY CHILDHOOD INVENTORY IN MATHEMATICS

- *The mathematics interview*
- *The content strands*
- *The developmental basis*

The mathematics interview

This assessment tool is designed for classroom teachers of Grades K-2 to identify the mathematical understandings of each child in their class. Teachers of Grade 3 may also wish to use this tool to assess students considered at risk in mathematics.

The assessment is conducted as a one-on-one interview. The child performs small mathematical tasks, and explains their thinking as they proceed. You observe and question the child. After you ask the child a question, the child answers orally. In many cases the child uses materials to find or show answers.

The content strands

This interview assesses the following content strands of mathematics: Counting, Numeration, Addition and Subtraction, Multiplication and Division, Geometry, Length Measurement, Time Measurement. These strands have research-based consistent paths of development in early childhood. This tool does not assess Function and Algebra, Statistics and Probability, or Mathematical Process, as consistent international research-based paths of early childhood development have not been plotted in these strands.

The strands used in this interview are separate. This is because children often have different combinations of understandings in mathematics. One child may be very strong in Numeration, but have only early understandings in Geometry. Another child of similar age may have strong understandings in Geometry, but be less advanced in Numeration.

The developmental basis

This assessment is underpinned by developmental sequences (see pages x-xx). These sequences of development typically occur slowly through ages 4 to 9, and will differ from one child to the next. Note that although these sequences are shown in "linear" and "boxed" formats, this is not to suggest that young children's mathematical learning is necessarily "linear" and/or "discrete'. A child who typically "counts on" to add small collections will often be able to instantly respond to some addition problems, such as simple "doubles": 2 + 2, and 3 + 3, and 4 + 4. Twomey Fosnot and Dolk use the term "landscape" to describe the complexity of children's learning pathways (2001).

Each strand of the mathematics interview looks at one area of early mathematics, and requires children to answer questions that reveal their developmental level of understanding. The focus is on the child's answers, and on the strategies used. For example, several different children might find the sum of 10 and 3, but one child may need to make a physical model of this and "count all," while another child might think of ten, and "count on" three more: "11, 12, 13 Another child may know that 10 and 3 makes 13 without having to count. Development is revealed by the use of increasingly abstract strategies.

For each developmental sequence in Counting, Numeration, Addition / Subtraction, Multiplication / Division, Geometry, Length Measurement and Time Measurement, see the following figures. Please remember that these sequences outline possible ranges of mathematical learning across several years of a child's life. The columns do not represent designated achievement for particular grade levels – many Kindergarten children will start the school year with understandings beyond the initial left hand column. In particular, the right hand columns of the sequences generally represent quite advanced understandings for Grade 2 children, and should not be read as goals for all Grade 2 students.

Early Childhood Counting Development

a. Beginning	*b. Early counting*	*c. Counts objects*	*d. Counts on by ones*	*e. Skip counts*	*f. Facility with counting*
These children are just beginning to learn the "counting by ones" sequence. They can recite small sections of the counting sequence. When counting objects, they are just beginning to coordinate one object with each number name.	These children are confident counting to ten. They can recite the counting sequence beyond ten, and can coordinate counting objects with number names to at least ten. They can use materials to show a given number less than ten.	These children are proficient at counting objects by ones. They know the counting sequence to at least thirty. They can coordinate counting objects with numbers names beyond twenty.	These children can mentally manipulate the "counting by ones" sequence. They can start the counting sequence at numbers other than one, and count on by ones. They can count backward by ones from ten.	These children are beginning to count in various intervals (skip count). They can skip count forward by twos, fives and tens. They can count backward by ones from twenty.	These children are proficient with counting in various intervals. They can skip count by various numbers, and can count by fives and tens starting at any number. They can count backward by ones from larger numbers.

Early Childhood Numeration Development

a. Beginning	*b. Understanding to 10*	*c. Exploring to 100*	*d. Understanding to 100*	*e. Understanding to 999*	*f. Understanding 1000 and beyond*
These children are just beginning to recognize numerals. They may name some numerals from zero to ten. They do not yet understand the order of these numbers.	These children know all numerals from zero to ten, and understand the correct order of these numbers.	These children are beginning to recognize and compare many two-digit numerals. They may confuse similar "sounding" numbers such as thirty and thirteen. They can tell the number that comes after a two-digit number.	These children are beginning to understand the place value of a two-digit number. They understand that ten ones are grouped to form one ten. They can order two-digit numbers, and can tell the number before a given two-digit number.	These children are able to recognize three-digit numerals. They have a growing understanding of the place value of three-digit numbers. They can compare and order numbers to 999.	These children can recognize and order numerals beyond 1000. They show a solid understanding of place value.

Early Childhood Addition and Subtraction Development

a. Beginning	*b. Count all strategies*	*c. Count on / count back strategies*	*d. Initial addition / subtraction strategies*	*e. Later addition / subtraction strategies*
These children are just beginning to understand the concepts of putting small collections together, or taking a small amount from a collection. They are not yet successful in finding sums or differences.	These children can successfully put two small collections together and find the sum. They can also find the difference after taking a small amount from a collection. They need to fully model, and count from one by ones to find their answers.	These children are beginning to use "short-cuts" to add two small collections, or to take a small amount from a collection. They only need part of a model, and either "count on" to find a sum, or "count back," "count down to," or "count up from" to find a difference.	These children are able to add and subtract small amounts by using thinking strategies, such as adding one or zero, using doubles, "turning around" the addends, combinations to ten, or adding ten. They are able to mentally add and subtract small numbers without making a model.	These children are able to extend their thinking strategies to include larger numbers, and more difficult calculations with small numbers. They add and subtract mentally, using thinking strategies such as fact families, "near doubles," build to ten, and adding nine.

Early Childhood Multiplication / Division Development

a. Beginning	*b. Full model / counts all*	*c. Full model / counts sets*	*d. Part model / counts sets*	*e. Multiplication strategies*	*f. Division strategies*
These children are just beginning to understand how to make small equal groups, and how to share a small amount equally. They are not yet successful in finding products or dividends.	These children can make small equal groups, and can share small amounts equally. They can find products by counting all objects by ones from one. They see a group of objects only as a collection of ones.	These children can make small equal groups, share small amounts equally, and divide a collection into groups of a given number. They use skip counting or repeated addition to find products. They see a group as "one unit" as well as a collection of ones.	These children need only part of a model to find products or dividends. They use skip counting or repeated addition to find products and dividends.	These children are able to multiply two numbers, using thinking strategies such as multiplying by tens, doubling, repeated addition, or "turning numbers around'. They are able to mentally multiply small numbers without making a model.	These children are able to divide a number equally, using thinking strategies such as dividing by tens, halving, and using repeated addition or subtraction. They are able to mentally divide small numbers without making a model.

Early Childhood Geometry Development

a. Beginning	*b. Whole shapes*	*c. Exploring shape parts*	*d. Understands shape parts*	*e. Shape families*
These children are just beginning to match very simple two-dimensional figures shown in conventional orientations.	These children can name and match simple two-dimensional figures shown in conventional orientations. They match shapes to common "prototypes," e.g. isoceles or equilateral triangles only. They attend to the whole of the shape, and do not yet describe any parts of a shape.	These children are beginning to attend to the parts of a shape, and typically describe the numbers of sides and corners a figure has, after counting these. They can visualize rotations of some simple shapes.	These children attend to many parts of a shape, and are able to fully describe a figure's properties. They recognize examples beyond common "prototypes," e.g. scalene and right triangles. They can visualize rotations and reflections of some simple shapes.	These children understand how shapes belong to families, e.g. a square is a special type of rectangle, and also a special type of rhombus. They attend to parallel lines, angle and perpendicular lines. They are able to visualize rota-tions and reflections of many shapes.

Early Childhood Length Measurement Development

a. Beginning	*b. Compares objects*	*c. Uses informal units*	*d. Uses formal units*	*e. Facility with formal units*
These children are just becoming aware of the attribute of length. They find the longest object perceptually.	These children can compare the length of two or more objects. They know how to align objects to compare their lengths. They can identify the longest and the shortest object.	These children can use informal units to measure and describe the length of an object. They understand how to repeatedly use a constant sized unit without gaps or overlaps.	These children can use one system of formal units to measure an object's length (either Standard or Metric, but not both). They are able to reasonably estimate a length.	These children can use both the Standard and the Metric systems to measure an object's length. They understand fractions of these units.

Early Childhood Time Measurement Development

a. Beginning	*b. Hours / days / months*	*c. Half hours / dates*	*d. Tells time*	*e. Facility with time*
These children are just beginning to read numerals on clock faces. They do not yet link these numerals to units of time.	These children can tell time to the hour. They know the days of the week and the months of the year.	These children can tell time to the half hour, and can find dates on a simple calendar.	These children can tell time to five-minute intervals, and can use a calendar to predict dates.	These children can calculate using hours and minutes.

4. INTERVIEWING

- *When?*
- *The student booklet*
- *Preparing to interview*
- *Interviewing*
- *Prompting*
- *Recording*
- *Interpreting*

When?

The assessment is presented in content strands that can be used at different times throughout the year. For example, it is useful to assess children's understanding of counting and numeration very early in the school year. Counting and numeration are embedded into much of the mathematics curriculum, such as measurement and data collection, so it is strategic to find out about children's abilities in these areas. Strands such as Multiplication / Division are usually taught later in the school year, so it is more appropriate to leave this assessment section until later in the year.

Ideally, the assessment would be taken before teaching the matching unit of mathematics. For example, the Addition / Subtraction assessment could be conducted before teaching a unit on Addition or Subtraction. Student results will then alert the classroom teacher to individual student differences in understanding.

Teachers may wish to use the entire interview with one or two students at the start of the school year. For a struggling student, this will provide information about their areas of need. For very able students, this will show how advanced their understandings are, and indicate how these children need to be further challenged. Some teachers may wish to use sections of the interview to gather information prior to parent/teacher interviews.

The student booklet

Each page in the student booklet has questions to ask the child, accompanied by prompts and teacher notes. There is space for recording the child's responses and for any additional comments, such as strategies the child used. Descriptors of development appear at the extreme right side of the interview page.

Each strand has several tabled sections. These tabled sections have questions that range from very early (Kindergarten) understandings, to Grade 3 understandings. When being interviewed, most children will complete only one, or possibly two of the tabled sections of the strand.

Preparing to interview

Each student requires a student record booklet. These should be kept at school as a part of the classroom teacher's student records. Each strand of the interview requires a small amount of materials. This list explains which materials need to be copied onto card stock, and which materials need to be collected from the classroom. For ease of management, it is best to prepare small bags of materials: one for each of Counting, Numeration, Addition / Subtraction, Multiplication / Division, Geometry, Length Measurement, and Time Measurement.

Have a small table area available where you will interview each child. Have the interview materials and student record booklets ready.

Each section of the interview takes about five to ten minutes per child. As teachers become practiced, this can be as quick as five minutes per child, meaning that a class of twenty children could be assessed in one of the mathematics strands within two fifty-minute periods.

Math Center time is a strategic time to interview children, as all other children will be productively engaged in independent mathematics tasks.

Interviewing

The assessment is best conducted by the child's classroom teacher. You will be able to observe and listen to each child as they answer mathematics questions. This provides insight into the child's understandings, and alerts you to areas the child does not yet understand.

Sometimes a resource teacher or the mathematics coach may conduct the assessment. In these cases, a subsequent meeting would be required to discuss each student's performance and understandings. ELL children should be interviewed in their first language by a teacher who is able to translate the questions and the child's responses.

You ask the child questions, and the child answers orally. In many cases the child uses materials to help find or show answers. Start at the section indicated by the child's grade level. Generally ask all questions in a tabled section, unless the child is struggling.

Stop if the child has difficulty with the first few questions in a section. If they have not attempted the previous tabled section, go back and work through these questions. If you are working with a particularly able student, you may wish to start them at the next tabled section.

Watch the child as they try to solve each question. Ask them to explain how they figured out their answers. Record their answers and any relevant observations, for example, "Using fingers when counting Progress to the next tabled section if the child has correctly answered all the questions. Continue until the child is having difficulty answering questions, or they are not using the indicated strategies.

Questions may be re-worded in the appropriate language as required to facilitate the child's understanding. A teacher who speaks the child's first language should interview ELL children. Children may answer in their first language. The interviewing teacher should translate, and record their responses in English.

Prompting

Sometimes children may not respond to a question. This may mean they do not understand what they are required to do. Use prompts to ensure the child understands what they have been asked to do. Suggested prompts are given for most questions. Rephrase the question in any way that will assist the child to continue. Do not, however, coach or teach the child during the interview. The purpose of the interview is to assess the child's current mathematical understandings.

If the child gives an incorrect answer, and you feel they are capable of answering the question, ask them to check, or try again. If the child initially gives an incorrect answer, and then corrects their own answer, accept and record their corrected answer.

Each child will reach a point where they cannot answer several consecutive questions. If you feel the child is very uncomfortable, cease the interview. Otherwise, ask just one or two more questions. Often a child's errors will provide information for instruction, for example, children who skip count by tens in the following manner: "Ten, twenty, thirty, forty, fifty, sixty, seventy, eight, ninety, one hundred, two hundred Remember to record the incorrect answers, as often these provide information for future instruction.

Recording

Record the child's answers, and their strategies in the student record booklets. Keep any recording discreet. Some children become anxious if they see check marks being recorded. If the child self-corrects, record the revised answer. Record any additional information in the "Comments" column. For example, when counting on from 57, the child may look around the room to locate numbers on the 100 chart.

Do not try to interpret the child's responses on the spot. Complete the assessment, and later look over the results.

Interpreting

Each tabled section has a rubric on the right hand side to find the child's development. Check the highest section of the mathematics strand that the child fully completed. The highest section that the child fully completed indicates the child's development within that strand.

Counting Assessment - SAMPLE

Student name: _____ **Date:** _____

MATERIALS: ***22 counters***

Grade K-1: Start here. Ask Q1-3.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Start at 1. Count aloud by ones. Stop the child at 30.	If necessary, prompt "1, 2, 3, like that."	1. 1-19	Then "ten-teen."	**a. Beginning** Small parts of Q1-3 correct: Oral counts less than 11 Counts less than 10 objects Unable to make small sets
2. Put out the pile of 22 counters. **Get 5 counters.** After this is done, push the 22 counters together again. Then: **Get 8 counters.**	Record the child's successful sequence, e.g. 1-19.	2. Makes sets: 5 8 Yes Yes		**b. Early counting** Parts of Q1-3 correct: Oral counts more than 10 Counts at least 10 objects Makes small sets
3. Use the pile of 22 counters. **Count all of these. Say the numbers as you count.**	Record the child's successful sequence, e.g., 1-12.	3. 1-12	Lost one-to-one correspondence after 12.	**c. Counts objects** All of Q1-3 correct: Oral counts by ones to 30 Counts more than 20 objects

Progress to Q4 if the child successfully answers all of Q1-3. Stop if the child has difficulty with any of Q4.

To interpret these results, check the rubric (a. Beginning, b. Early Counting, and c. Counts Objects) on the right hand side of the page. This child has completed "b. Early Counting" and could work toward "c. Counts Objects."

EARLY CHILDHOOD ASSESSMENT IN MATHEMATICS STUDENT RECORD BOOKLET

Children First Initiative

Student Name	Grade Level & Year	Grade Level & Year	Grade Level & Year	Grade Level & Year

Mathematics Strand	Date of Assessment							
Counting								
Numeration								
Addition & Subtraction								
Multiplication & Division								
Geometry								
Length Measurement								
Time Measurement								

Materials List

Materials printed in **bold** are in the Early Childhood Inventory in Mathematics Resource Cards. Other materials need to be collected from the classroom.

COUNTING

- 22 counters

NUMERATION

- **Numeral cards 0-10**
- **Two-digit numeral cards (17, 24, 42, 71)**
- Calculator
- Place value materials for tens and ones (8 tens and 15 ones of classroom material used to model place value: e.g., Base-10 blocks, or snap cubes in tens, or bundled pop sticks)
- Base-10 blocks for hundreds, tens and ones (3 hundreds, 8 tens and 15 ones)
- **Three-digit numeral cards (230, 509, 510, 570)**
- **Four- and five-digit numeral cards (5,046; 4,609; 30,718; 12,064)**

ADDITION AND SUBTRACTION

- 13 counters
- Index card for screening counters
- **Number sentence cards set A (initial facts: 5 + 2, 5 – 2, 12 – 1, 6 + 0, 6 – 0, 4 + 4, 6 + 6, 6 + 4, 7 + 3, 2 + 17, 37 + 10)**
- **Number sentence cards set B (later facts: 8 – 4, 10 – 7, 34 + 4, 40 + 40, 3 + 4, 6 + 7, 7 + 5, 13 – 4, 37 – 10, 37 + 9)**

MULTIPLICATION AND DIVISION

- **Picture of 5 cookie plates**
- 10 counters (all the same color)
- 12 snap cubes or unifix cubes joined together (all the same color)
- Index card for screening counters
- **Number sentence cards (multiplication set C: 4 x 5, 2 x 6, 3 x 10, 5 x 7; division set D: 30 ÷ 10, 12 ÷ 6, 20 ÷ 5)**

GEOMETRY

- **Shape cards A-E**
- Collection of mixed Pattern Blocks and counters: 7 green triangles, 2 orange squares, 3 red trapezoids, 4 blue rhombi, 1 yellow hexagon, 2 circular counters
- **Tangram puzzle pieces**

MEASUREMENT

Length

- 4 sticks (wooden meat skewers cut to these lengths: 5 cm, 10 cm, 18 cm, and 20 cm)
- 6 x 5 cm paper clips
- A 12-inch ruler with cm and inch graduated scales
- 1 sheet of legal-size paper (8.5 x 14 inches)

Time

- **Analog clock face with movable hands**
- **Digital clock time cards**
- Current classroom calendar

Counting Assessment

Student name: _____ **Date:** _____

MATERIALS: ***22 counters***

Grade K-1: Start here. Ask Q1-3.

Questions	Prompts, notes	Child's responses	Comments	Stage of learning
1. Start at 1. Count aloud by ones. Stop the child at 30.	If necessary, prompt "1, 2, 3, like that." Record the child's successful sequence, e.g., 1-19.	1. _____		**a. Beginning** Small parts of Q1-3 correct: Oral counts less than 11. Counts less than 10 objects. Unable to make small sets. **b. Early counting** Parts of Q1-3 correct: Oral counts more than 10. Counts at least 10 objects. Makes small sets. **c. Counts objects** All of Q1-3 correct: Oral counts by ones to 30. Counts more than 20 objects.
2. Put out the pile of 22 counters. **Get 5 counters**. After this is done, push the 22 counters together again. Then: **Get 8 counters.**		2. Makes sets: 5 8		
3. Use the pile of 22 counters. **Count all of these. Say the numbers as you count.**	Record the child's successful sequence, e.g., 1-12.	3. _____		

Progress to Q4 if the child successfully answers all of Q1-3. Stop if the child has difficulty with any of Q4.

Counting Assessment (continued)

Student name: _____ **Date:** _____

Grade 2: Start here. If the child has difficulty with any of Q4, revert to Q1-3.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
4a. Start at 57 and keep counting. Stop the child at 72. **4b. Count backward by ones. Start at 10.**	Record the child's successful sequences, e.g., 57-59, 10-7.	4a._____ b. _____		**d. Counts on by 1s** All of Q4 correct: Counts forward by ones from any two-digit number. Counts backward by ones from 10. **e. Skip counts** All of Q5 correct: Counts forward by 10s, 5s, 2s. Counts backward by ones from 20.
5a. Count backward by ones from 20. Stop the child at 9. **5b. Skip count by 10s**. Stop the child at 110. **5c. Skip count by 5s**. Stop the child at 50. **5d. Skip count by 2s**. Stop the child at 30.	If necessary, prompt, "10, 20, 30, like that." Stop if the child counts up by ones to say a skip counting sequence. Record the child's successful sequence, e.g., 20-16, 10-40, 2-12.	5a._____ b. _____ c. _____ d. _____		

Progress to Q6 if the child successfully answers all of Q4 and 5. Stop if the child has difficulty with any of Q6.

Counting Assessment (continued)

Student name: _____ **Date:** _____

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
6a. Skip count by 3s. Stop the child at 33. **6b. Skip count by 4s.** Stop the child at 48. **6c. Count backward by ones from 63.** Stop the child at 58.	Stop if the child counts up by ones to say a skip counting sequence. Record the child's successful sequences, e.g., 3-15, 63-60.	6a._____ 6b._____ 6c._____		**f. Facility with counting** All of Q6 and 7 correct: Counts forward by 3s, 4s. Counts backward by 1s from any two-digit number. Counts forward by 5s, 10s from any two-digit number.
7a. Start at 53. Skip count forward by 5s. Stop child at 88. **7b. Start at 78. Skip count forward by 10s.** Stop child at 118.	Record the child's successful sequences, e.g., 53-68, 78-98.	7a. _____ 7b. _____		

End of Counting Assessment section.

Numeration Assessment

Student name: _____ **Date:** _____

MATERIALS:

- ***Single-digit numeral cards 0-10***
- ***Two-digit numeral cards (17, 24, 42, 71)***
- ***Calculator***
- ***8 tens and 15 ones of classroom material used to model place value (e.g., Base-10 blocks, snap cubes in tens, or bundled pop sticks)***

Grade K, 1: Start here. Ask Q1-2.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Show the numeral cards 0-10. **Tell me the numbers you know.**	Show the cards out of counting order.	1. Known numerals: 0 1 2 3 4 5 6 7 8 9 10		**a. Beginning** Some of Q1 and 2 correct: No numerals known, or may know some numerals 0-10.
2. Take out the zero card. Give the child the numeral cards 1-10. **Put these numbers in counting order.** After this is done: **Which is the largest number? Which is the smallest number?**	Give the cards out of counting order. Cards can be sequenced from left to right, or right to left.	2. Orders _____ Largest _____ Smallest _____		**b. Understanding to 10** All of Q1, 2 correct: Knows all numerals 0-10 Orders numerals 1-10.

Progress to Q3 if the child successfully answers all of Q1-2. Stop if the child has difficulty with Q3 and 4.

Numeration Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Single-digit numeral cards 0-10***
- ***Two-digit numeral cards (17, 24, 42, 71)***
- ***Calculator***
- ***8 tens and 15 ones of classroom material used to model place value (e.g., Base-10 blocks, snap cubes in tens, or bundled pop sticks)***

Grade 2: Start here. If the child has difficulty with Q3 and 4, revert to Q1-2.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
3. Use the two-digit numeral cards. **Tell me the numbers you know.** After this is done: **Spread the cards out. Which is the largest number? Which is the smallest number?**	When spreading the cards, ensure they are out of order.	3. Known numerals: 17 24 42 71 Largest _____ Smallest _____		**c. Exploring to 100:** Identifies at least 5 of the numerals from Q3 and 4. Q5 incorrect. Knows some two-digit numerals, and the number after. **d. Understanding to 100** All of Q3, 4, 5 correct: Identifies and orders two-digit numerals. Able to model and explain the place value of a two-digit number. Tells the number before.
4. Give the child a calculator. **Type this number on the calculator: 25.** After this is done: **What number comes after this? What number comes before this?** Repeat for numbers 30, and 13.	Assist the child with starting and clearing the calculator as necessary.	4. Types numerals: 25 30 13 Number after / before: 26/24 31/29 14/12		
5. Use the classroom materials for tens and ones (6 tens and 15 ones). Put out 3 tens and 5 ones. **This is how I make 35: 3 tens and 5 ones.** Show the child the 42 card. **Use the materials to make 42.** After this is done: **Tell me how you figured that out.**	Counting by ones is incorrect. The child should use and count 4 tens and 2 ones. "10, 20, 30, 40, then 41, 42," is sufficient explanation.	5. Models 42 _____ Explains tens and ones _____ (Some reference to tens or counting by tens is necessary.)		

Progress to Q6 if the child successfully answers Q3-5.

Numeration Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Base-10 blocks (3 hundreds, 8 tens and 15 ones)***
- ***Three-digit numeral cards (230, 509, 510, 570)***

Stop if the child has difficulty with any of Q6 or 7.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
6. Use the three-digit numeral cards. **Tell me the numbers you know.**	If a child names the digits, e.g., "two, three, zero": **What is this number called?**	6. Known numerals: 230 509 510 570		**e. Understanding to 999** All of Q6, 7 and 8 correct: Knows and orders three-digit numerals. Able to model and explain the place value of a three-digit number. Models and explains ten and one hundred more, and ten and one hundred less (where regrouping is not required).
7. Give the child the three-digit numeral cards. **Put these numbers in order from smallest to largest.** After this is done: **How did you figure that out?**		7. Orders _____ Explains _____		
8a. Show the child the 230 card and the Base-10 blocks. **Use the blocks to show this number.** After this is done: **Explain what you did.**	Stop here if the child cannot build the model, or is counting by ones.	8a. Models 230 Explains hundreds, tens and ones _____		
8b. Continue to use the 230 card and the Base-10 block model of 230. **Show the number that is ten more than this with the blocks. What number is it?** If successful, continue to use the 230 model: **Show the number that is one hundred more than this. What number is it?** **Show the number that is ten less than this. What number is it?** **Show the number that is one hundred less than this. What number is it?**	Use 230 as the "starting" number each time. The child should add or remove blocks of appropriate value each time. The child need not predict each number at this level, but they must be able to "work out" the value of each new number after they adjust the model.	8b. Models 240. Says 240. Models 330. Says 330. Models 220. Says 220. Models 130. Says 130.		

Progress to Q9 if the child successfully answers all of Q6-8.

Numeration Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- *Calculator*
- *Four- and five-digit numeral cards (5,046; 4,609; 30,718; 12,064)*

Stop if the child has difficulty with any of Q9.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
9. Use the four- and five-digit numeral cards. **Tell me the numbers you know.**	If a child uses an alternative name such as "thirty-seven hundred and eighteen": **What else is this number called?**	9. Known numerals: 5,046 4,609 30,718 12,064		**f. Understanding 1000 and beyond** All of Q9, 10, 11 correct: Knows and orders numerals up to five digits. Understands the place value of a four-digit number.
10. Give the child the four- and five-digit numeral cards. **Put these numbers in order from smallest to largest.** After this is done: **How did you figure that out?**		10. Orders _____ Explains _____		
11. Give the child a calculator. **Type this number on the calculator: 7,415.** After this is done: **What number is ten more than this? How do you know?** **What number is one hundred more than this? How do you know?** **What number is ten less than this? How do you know?** **What number is one hundred less than this? How do you know?**	Use 7,415 as the "starting" number each time. The child should predict each new number, and explain which digit changes. Assist the child with starting and clearing the calculator as necessary.	11. Types 7,415 _____ 7, 425: Ten more _____ 7, 515: One hundred more _____ 7, 405: Ten less _____ 7, 315: One hundred less _____		

End of Numeration section.

Addition and Subtraction Assessment

Student name: _____ **Date:** _____

MATERIALS:

- ***13 counters***
- ***Index card for screening the counters***

Grade K, 1: Start here.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Use 10 counters. Show the child 6 in one set and 4 in another set. **There are 6 counters here and 4 here.** Cover the 6 counters with a card. **How many counters in all?** After the child answers: **How did you figure that?**	If child cannot do this, remove the card and repeat the question. Let them use the counters.	1. ____ Solves with partial model or Needed full model Strategy: Known fact Count on Count all		**a. Beginning:** Unable to solve Q1, 2, 3 by any method. **b. Count all strategies:** Needs the full model to solve any of Q1, 2, and 3. Strategy used: counts all (by ones from one). **c. Count on/count back strategies:** Needs only part of the model to solve all of Q1, 2 and 3. Strategy used: counts on for Q1 and 2, count back/down/up for Q3, or known facts.
2. Put the 10 counters together. **Here are 10 counters. Imagine we put 3 more counters there. How many counters would that make in all?**	If the child cannot figure this mentally, give them three more counters to use.	2. ____ Solves with partial model or Needs full model Strategy: Known fact Count on Count all		
3. Put out 9 counters. **There are 9 counters here.** Now cover the counters with a card. **If we take 3 counters away, how many counters would be left?** After the child answers: **How did you figure that?**	If child cannot do this, remove the card and repeat the question. Let them use the counters.	3. ____ Solves with partial model or Needs full model Strategy: Known fact Count back, down or up Counts by ones from one		

Grade K, 1: Progress to Q4 if the child solves Q1-3 with only part of the model shown, and using count on, count back/down/up strategies, or known facts.

Addition and Subtraction Assessment

Student name: _____ **Date:** _____

MATERIALS: ***Number sentence cards set A, B***

Grade 2: Start here. Stop whenever the child has difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
4. Use number sentence cards set A. Show each number sentence and read each one aloud to the child. **I'm going to read some math questions to you. Try to figure these in your head.** **a)** **5 + 2** **5 – 2** **12 – 1** (Add/subtract 1,2) **b)** **6 + 0** **6 – 0** (Add/subtract 0) **c)** **4 + 4** **6 + 6** (Doubles) **d)** **2 + 17** (Turn around strategy) **e)** **6 + 4** **7 + 3** (Sums to 10) **f)** **37 + 10** (Adding 10)	Immediate correct responses suggest the child has used initial addition or subtraction strategies. Use of fingers, or counting by ones indicates the child is using counting strategies. If at any time the child gives a delayed answer: **How did you figure that?**	a) _____ b) _____ c) _____ d) _____ e) _____ f) _____		**d. Initial addition/subtraction strategies:** Solves 10 of the 11 number sentences from Q4 using initial addition/subtraction strategies.

Progress to Q5 if the child successfully answers 10 examples from Q4 using initial addition/subtraction strategies. Stop whenever the child has difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
5. Use number sentence cards set B. Show each number sentence and read each one aloud to the child. **I'm going to read some math questions to you. Try to figure these in your head.** **a)** **8 – 4** **10 – 7** (Subtraction: fact families) **b)** **34 + 4** **40 + 40** (Using larger numbers) **c)** **3 + 4** **6 + 7** (Doubles plus/minus one) **d)** **7 + 5** **13 – 4** (Make to ten or compensation) **e)** **37 – 10** (Subtracting 10) **f)** **37 + 9** (Adding 9)	Immediate correct responses suggest the child has used later addition or subtraction strategies. Use of fingers, or counting by ones indicates the child is using counting strategies. If at any time the child gives a delayed answer: **How did you figure that?**	a) _____ b) _____ c) _____ d) _____ e) _____ f) _____		**e. Later addition/subtraction strategies:** Solves all 10 number sentences from Q5 using later addition/subtraction strategies.

End of Addition and Subtraction section.

Multiplication and Division Assessment

Student name: _____ **Date:** _____

MATERIALS:

- *Picture of 5 cookie plates*
- *10 counters (all the same color)*
- *12 snap cubes (all the same color)*
- *Index card for screening the counters*

Grade K: The Multiplication/Division section is optional. If you wish to assess K children, start here. Grade 1, 2: Start here.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Show the picture of 5 cookie plates. **Imagine there are 2 cookies on each plate. How many cookies would this be in all? Tell me how you figured that.** If child cannot do this mentally, give them counters and let them build a model on the plates. After this is done: **How many cookies is this in all?**	Look for skip counting, repeated addition or using known facts. If the child counts by ones from one: **Can you do that a quicker way?**	1. Answer _____ Solves with partial model or Needs full model Strategy: Known fact Skip count Counts by ones		**a. Beginning:** Unable to solve any, or solves just one of Q1, 2, and 3 using any method. **b. Full model/counts all:** Two of Q1-3 correct. Needs the full model for every question. Strategy used: counting all from one (can't yet see a modelled group as "one unit"). **c. Full model/counts sets:** All of Q1-3 correct. Needs the full model to solve any of the questions. Strategy used: skip counting or known facts (sees a modelled group as "one unit"). **d. Part model/counts sets:** All of Q1-3 correct. Needs only part of the model for every question. Strategy used: skip counting or known facts.
2. Use 8 counters. **Here are 8 counters.** Now cover them with a card. **Imagine we share the counters fairly between you and me, so we each get the same amount. How many counters would we each get? How did you figure that?** If child cannot do this mentally, give them 8 counters and let them share the counters out.	Look for skip counting, repeated subtraction or using known facts.	2. Answer _____ Solves with partial model or Needs full model Strategy: Known fact Skip count Counts by ones		
3. Show a stick of 12 snap cubes. **Here are 12 cubes. Imagine the 12 cubes are put into groups of 3. How many groups of 3 would there be? Tell me how you figured that.** If the child cannot do this mentally, let them break the stick into groups of 3. Then: **How many groups of 3 are there?**	The child might use fingers for partial modelling, e.g., put up one finger each time they skip count by threes. Answering "12" indicates the child is still fully modeling and counting all.	3. Answer _____ Solves with partial model or Needs full model Strategy: Known fact Skip count Counts by ones		

Progress to Q4 if the child successfully solves Q1-3 with only part of the model shown, and using skip counting, repeated addition, or known facts.

Multiplication and Division Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Multiplication number sentence cards set C***
- ***Division number sentence cards set D***

Stop whenever the child has difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
4. Use the multiplication number sentence cards set C. Show and read each number sentence aloud to the child. **I'm going to read some math questions. Try to figure these in your head.**	Quick correct responses suggest the child has used multiplication strategies. If at any time the child gives a delayed answer: **How did you figure that?**			**e. Multiplication strategies:** Solves all of Q4 by multiplication strategies.
a) 3 x 10 (Tens strategy)	Think, e.g., 3, add zero	a) _____		
b) 2 x 6 (Doubling strategy)	Think, e.g., six plus six	b) _____		
c) 4 x 5 (Repeated addition strategy)	Think, e.g., 5, 10, 15 20; or 5 + 5 + 5 + 5	c) _____		
d) 5 x 7 (Turn around strategy)	Think, e.g., 7 times 5	d) _____		

Progress to Q5 if the child successfully solves all of Q4 using known facts. Stop whenever the child has difficulty.

Multiplication and Division Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Multiplication number sentence cards set C***
- ***Division number sentence cards set D***

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
5. Use the division number sentence cards set D. Show and read each number sentence aloud to the child. **I'm going to read some math questions. Try to figure these in your head.**	Quick correct responses suggest the child has used division strategies. If at any time the child gives a delayed answer: **How did you figure that?**			**f. Division strategies:** Solves all of Q5 by division strategies.
a) 30 ÷ 10 (Tens)	Think, e.g., 30, remove zero	a) _____		
b) 12 ÷ 6 (Halving)	Think, e.g., 6 + 6 = 12, so 12 – 6 = 6	b) _____		
c) 20 ÷ 5 (Repeated addition or subtraction)	Think, e.g., 5 + 5 + 5 + 5 = 20, so there are 4 fives.	c) _____		

End of Multiplication and Division section.

Geometry Assessment

Student name: _____ **Date:** _____

MATERIALS:

- ***Shape cards A-C***
- ***Small collection of shapes (Pattern Blocks and circular counters: 7 green triangles, 2 orange squares, 3 red trapezoids, 4 blue rhombi, 1 yellow hexagon, 2 circular counters)***

Grade K, 1: Start here.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Use shape card A and the collection of shapes. Point to each shape on the card and say: **Find the block that has this shape.** After this is done, point to the circle, square and triangle, and ask: **What is this shape called?**	The child does not need to name the rhombus, hexagon or trapezoid.	1. Matches: All shapes matched ____ Circle named ____ Square named ____ Triangle named ____		**a. Beginning:** *Any of Q1,2 incorrect.* Not yet able to recognize and name shapes shown in "conventional" orientations. **b. Whole shapes:** All of Q1, 2 correct. Recognizes and names shapes shown in "conventional" orientations.
2. Use shape card B. Point to each shape and say: **Look around the room. Tell me some things that have that shape.**	The child does not need to name the rectangles used in Q2.	2. Circular objects___ Rectangular objects ___		

Grade K, 1: Progress to Q3 if the child successfully answers all of Q1-2. Stop if the child does not give any shape parts for Q3.

Geometry Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Shape cards A-C***
- ***Small collection of shapes (Pattern Blocks and circular counters: 7 green triangles, 2 orange squares, 3 red trapezoids, 4 blue rhombi, 1 yellow hexagon, 2 circular counters)***

Grade 2: Start here. If the child cannot answer Q3 with information about parts of shapes, revert to Q 1-2.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
3. Use shape card C. Point to the triangle and the square**. How are these shapes different? What do they have that is different?** After the child has explained, repeat for the square and the pentagon, and then the triangle and the pentagon.	The child should notice and describe at least the shapes' sides and corners. Vertices may be called "corners" or "points." Sides may be called "lines" or "edges." The child does not need to name the pentagon. Have the child build the shape beside the hexagon block (not on top of it).	3. Triangle description: 3 sides __ 3 corners ____ Square description: 4 sides __ 4 corners ____ Pentagon description 5 sides __ 5 corners ____		**c. Exploring shape parts:** Number of sides and corners noticed and described for shapes in Q3. Gives at least two solutions for Q4. Identifies some parts of shapes, and visualizes simple orientation of shapes.
4. Show the child the Pattern Blocks. Point to the yellow hexagon. **What shapes could you put together to build this shape?** Let the child use selected blocks to build a hexagon. If successful, ask: **Can you do that using different shapes?**		4. Builds shape _____ Shapes used _____ A different solution ____ Shapes used _____		

Progress to Q5 if the child successfully answers all of Q3-4. For Q3, the child must refer to the numbers of sides and corners of the shapes.

Geometry Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- *Shape cards D, E*
- *Tangram puzzle pieces*

Stop if the child has any difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
5. Use shape card D. **Which shapes are triangles?** After the child has done this, ask: **How can you tell?** After the child has explained this, point to a "non-triangle" and ask: **Why is this not a triangle?** Repeat for the other "non-triangles" if necessary.	The child should explain in his or her own words. Listen for statements such as, "It has to join up," or "It can't have any curvy sides." The squares will be different sizes.	5. Some triangles found. Some "non-triangles" called triangles. All triangles found. A triangle has: 3 sides ___ 3 corners ___ Straight sides _____ A closed shape _____		**d. Understands shape parts:** Refers to closed shape, straight sides, 3 sides, 3 corners for triangles. Correctly identifies all triangles for Q5. Makes three-piece square for Q6. Identifies many parts of shapes, and visualizes rotations and reflections of some simple shapes.
6. Use the tangram puzzle pieces. Spread out the seven pieces. **Can you put three of these pieces together to make a square?** If successful, then separate the pieces and say: **Can you put four of these pieces together to make a square?**	The child should explain in his or her own words. Listen for statements such as, "It has to have square shaped corners," or "It can't have any slanting lines."	6. "Three-piece" square: Builds square _____ "Four-piece" square: Builds square _____		**e. Shape families:** Refers to closed shape, straight sides, 4 sides, 4 corners, opposite sides of equal length, right angle corners for rectangles. Correctly identifies all rectangles (including square) for Q7. Makes both squares for Q6. Understands how shapes relate in "families," and able to visualize rotations and reflections of many shapes.

Geometry Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- ***Shape cards D, E***
- ***Tangram puzzle pieces***

Stop if the child has any difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
7. Use shape card E. **Which shapes are rectangles?** After the child has done this, ask: **How can you tell?** After the child has explained this, point to a "non-rectangle" and ask: **Why is this not a rectangle?** Repeat for the other "non-rectangles" if necessary.		7. Some rectangles found: Some "non-rectangles" called rectangles. All rectangles found (including square). A rectangle has: 4 sides ___ 4 corners ___ Straight sides _____ A closed shape _____ Right angle corners _____ Opposite sides equal length _____		

End of Geometry section.

Length Measurement Assessment

Student name: _____ **Date:** _____

MATERIALS:

- *4 sticks (wooden meat skewers cut to these lengths: 5 cm, 10 cm, 18 cm, and 20 cm)*
- *6 x 5 cm paper clips*
- *12-inch ruler with cm and inch graduated scales*
- *1 sheet of legal-size paper (8.5 x 14 inches)*

Grade K, 1: Start here.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1. Show the 4 sticks in a random arrangement. **Which is the longest? Which is the shortest?** If the child does not align the sticks to compare, say: **Show me how you can check.**	If the child does not understand "shortest," ask for the "smallest."	1. Longest _____ Shortest _____ Aligns to compare _____		**a. Beginning** Any of Q1 incorrect: Not yet able to compare the length of two or more objects. **b. Compares objects** Q1 correct, any of Q2 incorrect: Compares the length of two or more objects. **c. Uses informal units** All of Q1, 2 correct: Measures and describes the length of an object using informal units.
2. Use the longest stick (20 cm) and the 6 paper clips. **Measure this stick using the paper clips.** After this is done: **How long is the stick?** If the child just says "4," ask: **4 what?** If they say "4 inches," point to a clip and ask: **Is that an inch? What is this?**	The child should align 4 clips lengthwise without any gaps or overlaps. If the child does not understand, align one clip beside the stick and say, "Like this."	2. Use of non standard units _____ Measurement given ___ Suitable units given ___ The child must say suitable units (e.g., "4 clips," or "4 pins").		

Grade K, 1: Progress to Q3 if the child successfully answers Q2. Stop if the child has difficulty.

Length Measurement Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- *4 sticks (wooden meat skewers cut to these lengths: 5 cm, 10 cm, 18 cm, and 20 cm)*
- *6 x 5 cm paper clips*
- *a 12-inch ruler with cm and inch graduated scales*
- *1 sheet of legal-size paper (8.5 x 14 inches)*

Grade 2: Start here. If a child has difficulty with Q3a, go back to Q 1 and 2.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
3a. Use the longest stick (20 cm). **About how long is this stick in inches?** After the child gives an estimate, hand them a ruler. **Now measure the stick in inches.** Then ask: **How long is the stick?**	Estimates for Q3 should be reasonable (between 6 and 10 in, between 15 and 25 cm) The child should say the appropriate unit (inch or centimeter).	3a. Inch estimate _____ Correct use of ruler _____ Measurement _____		**d. Uses formal units:** Either Q3a and 3b correct, but not both. Measures the length of an object using one system of formal units (Metric or Standard). **e. Facility with formal units:** Q3a, 3b and Q4 correct. Measures accurately using Metric and Standard units of measure.
3b. Continue to use the longest stick. **About how long is the stick in centimeters?** After the child gives an estimate: **Now measure the stick in centimeters.** Then ask: **How long is the stick?**		3b. Estimate in cm _____ Measurement _____		
4. Use the sheet of Legal size paper and the ruler. **Measure the shorter side in inches.** Then: **Measure the longer side in inches.**	The width given should be 8½ inches. The length given should be 14 inches, or 1 foot and 2 inches.	4. Width ___ Length _____		

End of Length Measurement section.

Time Measurement Assessment

Student name: _____ **Date:** _____

MATERIALS:

- ***Analog clock face with movable hands***
- ***Digital clock time cards***
- ***Current classroom calendar***

Grade 1: Start here. Grade K: The Time Measurement section is optional. If you wish to assess K children, start here.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
1a. Say the days of the week. **1b. Say the months of the year.**	These can be said in any order.	1a. All days _____ 1b. All months _____		**a. Beginning:** Any of Q1-4 incorrect. Not yet able to tell time to the hour, know days of the week, and months of the year. **b. Hours/days/months** All of Q1-4 correct: Tells time to the hour, knows days of the week and months of the year.
2. Tell me something you do in the morning. Repeat this question for afternoon, and for night.	The child should say appropriate activities.	2. M _____ A _____ N _____		
3a. Use the analog clock face. **Tell me these times.** Show the child 3 o'clock, then 10 o'clock. **3b**. Use the digital clock cards (5:00, 9:00). **Tell me these times.** Then point to the 9:00 card and give the child the analog clock face. **Make this time on the clock.**	The clock hands should be correctly placed (not reversed).	3. _____ _____ 3b. _____ _____ Shows 9:00 _____		

Progress to Q4 if the child successfully answers Q1-3. Stop if the child has difficulty.

Time Measurement Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS:

- *Analog clock face with movable hands*
- *Digital clock time cards*
- *Current classroom calendar*

Grade 2 start here. If the child has difficulty with any of Q4, revert to Q1-3.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
4. Use the analog clock face. **Tell me these times.** Show the child 3:30 and 8:30. For each time, say: **Tell something that you would be doing at this time.**	Either "three-thirty" or "half past three" can be used. The child should say appropriate activities.	**4.** ____ ____ Activity _____ _____		**c. Half hours/dates** All of Q4-6 correct: Tells time to the half hour, and reads a calendar.
5. Use the digital clock card (7:30). **Tell me this time.** Then give the child the analog clock face. **Make this time on the clock.**	The clock hands should be correctly placed (not reversed).	5. _____ Shows 7:30 _____		
6. Use the class calendar. **Look at the calendar. What is today's date? What will tomorrow's date be? What was yesterday's date? How can you tell?**	Listen for the day, date and month. If any are missed, ask: **And what ___ is it?**	6. Today _____ Tomorrow _____ Yesterday _____		

Progress to Q7 if the child successfully answers Q4-6.

Time Measurement Assessment (continued)

Student name: _____ **Date:** _____

MATERIALS

- *Analog clock face with movable hand*
- *2:50 digital clock time card*
- *Current class calendar*

Stop if the child has difficulty.

Questions	*Prompts, notes*	*Child's responses*	*Comments*	*Stage of learning*
7a. Use the analog clock face. **Tell me this time.** Show the child 11:15. **7b.** Use the 11:15 time. **Imagine one more hour goes by. What time will it be then?** **7c.** Use the 11:15 time**. Imagine ten more minutes go by. What time will it be then?**	It is acceptable for the child to count by 5s to find the "minutes past."	7a. 11:15 _____ 7b. 12:15 _____ 7c. 11:25 _____		**d. Tells time** At least four parts of Q7-9 correct: Tells time to five minute intervals, and uses a calendar to find various dates. **e. Facility with time** All of Q7-9 correct: Tells time accurately.
8a. Show the 2:50 digital clock card and give the child the analog clock face. **Make this time on the clock.** **8b.** Use the 2:50 time. **Imagine fifteen more minutes go by. What time will it be then?** **8c.** Use the 2:50 time. **Imagine it is one hour earlier. What time would that have been?**	The clock hands should be correctly placed (not reversed).	8a. Shows 2:50 _____ 8b. 3:05 _____ 8c. 1:50 _____		
9a. Use the class calendar. **Look at the calendar. What will the date be one week from now?** After this is done: **How did you know?** **9b. What will the date be two weeks from now?** After this is done: **How did you know?**	If either of these dates bridge to the next month, show the child the last day of the month, e.g., today's date, 21st October, show the child the day that will be October 31st.	9a. Next week's date _____ 9b. Two weeks from today _____		

End of Time Measurement section.

5. IMPLICATIONS FOR INSTRUCTION

- *Identifying children's current understandings*
- *Children's next learning steps*
- *Differentiating instruction*
- *Confident students*
- *Struggling students*

Identifying children's current understandings

After assessing the children's understandings within a content strand of mathematics, you have a "picture" of the variation within your classroom. The class can be "mapped." An example is shown below.

Grade 1. Write your students under the most advanced stage they have completed.

Early Childhood Counting Development

a. Beginning	*b. Early counting*	*c. Counts objects*	*d. Counts on by ones*	*e. Skip counts*	*f. Facility with counting*
Small sections of the counting sequence.	Counts to ten, makes small sets. Bryan Ashanti Aimilly Ping Jefferson	Counts objects beyond 20. Krystal Shazarae David Ben Tyrique Ruth Christopher Blair Sincere Fatima Jose Zoe	Counts on by ones from a two-digit number. Davon Jesus Tyshawn Alana Keisha Jorge Brittany	Skip count forward by twos, fives and tens.	Skip count by various numbers.

Children's next learning steps

When you map your children's development within a mathematics strand, you are able to see the "next learning step" for each group of students. The example below uses arrows to indicate the level of instruction that each group of children could now work toward. The five children who can count to ten need experiences counting larger collections. The thirteen children who can count object accurately are ready to learn to count on by ones from numbers other than one. The seven children who can count on from any two-digit number are ready to learn skip counting. If instruction is differentiated accordingly, then all three groups of children have the opportunity to build their Counting understandings.

Early Childhood Counting Development

a. Beginning	*b. Early counting*	*c. Counts objects*	*d. Counts on by ones*	*e. Skip counts*	*f. Facility with counting*
Small sections of the counting sequence.	Counts to ten, makes small sets. Bryan Ashanti Aimilly Ping Jefferson	Counts objects beyond 20. Krystal Shazarae David Ben Tyrique Ruth Christopher Blair Sincere Fatima Jose Zoe	Counts on by ones from a two-digit number. Davon Jesus Tyshawn Alana Keisha Jorge Brittany	Skip count forward by twos, fives and tens.	Skip count by various numbers.

Differentiating instruction

It is strongly advocated that a workshop lesson be taken (mini-lesson, student activity, reflection time). This lesson structure has a flexibility that lends itself to adaptation for different children's learning needs.

Four major teaching strategies are advocated for you to use. These will enable differentiated instruction for students identified as needing challenge or modification:

1. **Modifying lessons**
2. **Open-ended lessons**
3. **Problem posing**
4. **Small group instruction**

1. Modifying lessons

Using your mathematics program, plan your lesson to meet grade level requirements. It is likely that this will be appropriate for the "middle" group of your students. Then think of more challenging requirements and less demanding requirements within the same lesson. These three "levels" of instruction can be matched to the students" learning needs. Number lessons can generally be made more challenging by increasing the number range, and by working mentally rather than with concrete or visual materials. Number lessons can generally be made simpler by decreasing the number range, and by having children use more concrete and visual materials. Geometry lessons can be generally be made more challenging by using more complex shapes or more complex transformations, and made simpler by using simpler shapes or simpler transformations. Three examples using this teaching strategy follow:

Grade: Kindergarten **Lesson focus: Counting objects, making and ordering sets** **Differentiation: Have different number limits for each level (1-4, 1-7, 1-10)**		
Pre level	***Mid level***	***Post level***
Children use snap cubes to build a staircase 1, 2, 3, 4 for a plastic teddy. Count forward up the steps. Draw and number the steps.	Children use snap cubes to build a staircase 1, 2, 3, 4, 5, 6, 7 for a plastic teddy. Count forward up the steps. Draw and number the steps.	Children use snap cubes to build a staircase 1-10 for a plastic teddy. Count forward up the steps, and backward down the steps. Draw and number the steps.

Grade: 1 Lesson focus: Adding to 10 Differentiation: Use different materials at each level (dot cards, a number chart, mentally)		
Pre level	***Mid level***	***Post level***
Children play a card game matching pairs of numbers that sum to ten. The cards have dots as well as numbers to enable children to count to 10.	Children play a card game matching pairs of numbers that sum to ten. The cards show numerals only. Children refer to a chart showing number sentences that add to 10 if needed.	Children play a card game matching pairs of numbers that sum to ten. The cards show numerals only. Children work mentally.

Grade: 2 Lesson focus: Adding coins Differentiation: Use different denominations of coins for each level		
Pre level	***Mid level***	***Post level***
Children select several mixed coins from a bag and find the total value. Repeat many times. Use nickels and pennies.	Children select several mixed coins from a bag and find the total value. Repeat many times. Use dimes, nickels and pennies.	Children select several mixed coins from a bag and find the total value. Repeat many times. Use quarters, dimes, nickels and pennies.

2. Open-ended lessons

An open-ended lesson is investigative in nature, and has a range of answers. All students can do the task, but their output will differ considerably. Strong students may complete many examples, while struggling students may complete only a few examples. Several examples of this teaching strategy follow.

Grade K Lesson focus: Length Measurement – "shorter than" Differentiation: Through the number of examples found
Each child is given a length of string. They find and draw objects in the classroom that are shorter than their length of string. Children should find as many examples as they can.

Grade 1 Lesson focus: Adding small amounts of money Differentiation: Through the number of examples drawn and added
Seven or eight real toys are displayed with price tags between 1 cent and 10 cents. Each child chooses two toys to "buy." They draw these and find the total cost of each pair of toys. Children should find as many examples as they can.

Grade 2 Lesson focus: Geometry Differentiation: Through the number of different examples recorded
Each child has four congruent right triangles. They fit the four triangles together to make as many different shapes as possible. (Triangles should be joined along "same length" sides.) They should sketch and describe each different shape.

3. Problem posing

Following a mini-lesson where a mathematical concept is modeled and discussed, you can ask students to then pose their own similar problems. This is very like the practice "innovating on the text'; used in literacy teaching. For example, a lesson might require children to explore adding two numbers, and then "turning the numbers around" to see if the order of addition affects the sum. Children could write their own number pairs to use in addition sentences; some children using numbers below ten, some using numbers between ten and twenty, and others using numbers beyond twenty. Children should be encouraged to work in number ranges and pose problems that are challenging to them: not too easy and not too difficult.

4. Small group instruction

When common learning needs are identified, you can plan to take a small group of children for instruction, while other students work independently. This requires the class to be able to continue at a set task for ten to fifteen minutes without interrupting you. Regular use of math centers is an ideal way to provide you with time to instruct a small group. Your classroom should have a range of accessible and engaging mathematics center tasks, and all children should understand how to collect, set up, use and pack away the center tasks. It is recommended that you try to rotate your teaching groups, so that over time there is contact with all learners, not just children experiencing difficulty with mathematics.

Confident students

When trial school teachers assessed their students, between 10 and 20% of the class were consistently performing above the grade level. (See "Some findings from the New York City classroom trials.") If these teachers had taught the grade-level curriculum without differentiation, these children would be going over mathematics they already understood at the start of the school year. This would have meant they would not be exposed to any new learning in counting, numeration or addition and subtraction.

Classroom teachers were able to build upon these children's learning by using the teaching strategies of modifying lessons, open-ended lessons, problem posing, and small group instruction. In some cases, more advanced homework was also given.

TRAN

Tran was a very capable Grade 1 student. His teacher was surprised to find that at the start of the school year, he could already read and model three digit numbers. When taking the mathematics lesson, she consistently modified lessons for Tran, letting him use larger numbers. She also encouraged him to pose and solve his own arithmetic problems, and arranged for Tran to have a more difficult homework book.

Struggling students

When trial school teachers assessed their students, between 5 and 10% of the class were consistently performing below the grade level (see "Some findings from the New York City classroom trials'). If these children's classroom teachers had taught the grade-level curriculum without differentiation, these children would be expected to learn mathematics that was too advanced for them. This would have meant they would either become quite confused, or have significant "gaps" in their learning.

Classroom teachers were able to successfully support these children's learning by using the teaching strategies of modifying lessons, open-ended lessons, problem posing, and small group instruction. In some cases, children were given small daily mathematics tasks to build confidence and "catch up" certain mathematical understandings.

OMAR

Omar was a Grade 2 student. Early in the school year, his teacher found that he had trouble counting between 10 and 20, and recognizing these numerals. She organized for another (more confident) student to work with Omar each day. Together they counted twenty objects, made sets of objects using the numerals 11 to 20, and ordered the numerals from 11 to 20. They did these tasks for five minutes while children had their snack each day. After three weeks of daily practice, Omar could proudly count to 20 and knew the numerals from 11 to 20.

5. RECORD KEEPING PAGES

- *Using the record keeping pages*
- *Class summary pages*
- *Class checklist pages*
- *Individual student record*

Using the record keeping pages

This section has a range of record keeping sheets that you may find useful. The Class Summary pages can be used to record the class understandings on one strand of mathematics. These provide strong direction for you to plan mathematics lessons that cater for different groups of children. The Class Checklist pages can be used for ongoing assessment during mathematics lessons. They are aligned to the same developmental sequences as the Class Summary pages. The Individual Student Record page can be used to profile one child's understandings across the various mathematics strands.

Class Summary: Early Childhood Counting Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Early counting*	*c. Counts objects*	*d. Counts on by 1s*	*e. Skip counts*	*f. Facility with counting*
Small sections of the counting sequence.	Counts beyond ten, makes small sets.	Counts objects beyond twenty.	Counts on by ones sequence from a two-digit number.	Skip count forward by twos, fives and tens.	Skip count by various numbers.

Class Summary: Early Childhood Numeration Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Understanding to 10*	*c. Exploring to 100*	*d. Understanding to 100*	*e. Understanding to 999*	*f. Understanding 1000 and beyond*
May recognize some numerals from 0-10.	Recognizes and orders numerals from 1-10.	Recognizes many two-digit numerals.	Understands "tens and ones" place value, orders two digit numerals.	Recognizes, orders and understands place value of three-digit numerals.	Recognizes, orders and understands place value of numerals beyond 1000.

Class Summary: Early Childhood Addition and Subtraction Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Count all strategies*	*c. Count on / count back strategies*	*d. Initial addition / subtraction strategies*	*e. Later addition / subtraction strategies*
Not yet successful in combining small collections to find sums.	Combines small collections and takes away small amounts to find sums or differences.	Counts on to find a sum, counts back, counts down to, or counts up from to find a difference.	Uses simpler thinking strategies to add and subtract (adding one or zero, using doubles, turning around the addends, sums to ten, adding ten).	Use later thinking strategies (fact families, extending to larger numbers, near doubles, make to ten, adding nine).

Class Summary: Early Childhood Multiplication / Division Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Full model / counts all*	*c. Full model / counts sets*	*d. Part model / counts sets*	*e. Multiplication strategies*	*f. Division strategies*
Not yet successful in making equal groups and sharing to find products or dividends.	Makes small equal groups, shares small amounts equally. Counting by ones.	Groups, shares, divides collections. Sees a group as one unit.	Uses skip counting or repeated addition to find products and dividends.	Uses thinking strategies to multiply (by tens, doubling, repeated addition, turning around).	Uses thinking strategies to divide (by tens, halving, repeated addition or subtraction).

Class Summary: Early Childhood Geometry Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Whole shapes*	*c. Exploring shape parts*	*d. Understands shape parts*	*e. Shape families*
Beginning to match very simple 2-D figures in standard orientations, not yet naming shapes.	Names and matches simple 2-D figures shown in standard orientations. Not yet describing shape parts.	Beginning to attend to shape parts, can visualize rotations of simple shapes.	Describes a figure's properties, visualizes rotations and reflections of simple shapes.	Understands how shapes belong to families, attends to lines and angles, visualizes rotations and reflections of many shapes.

Class Summary: Early Childhood Length Measurement Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Compares objects*	*c. Uses informal units*	*d. Uses formal units*	*e. Facility with formal units*
Becoming aware of length, finds the longest object perceptually.	Compares the length of two or more objects.	Uses informal units to measure and describe the length of an object.	Uses one system of formal units to estimate and measure an object's length (either Standard or Metric).	Uses both the Standard and the Metric systems to measure an object's length, understands fractions of units.

Class Summary: Early Childhood Time Measurement Development

Grade Level _____

Write the first names of your students under the most advanced stage of learning they have completed.

a. Beginning	*b. Hours / days / months*	*c. Half hours / dates*	*d. Tells time*	*e. Facility with time*
Know some days and months, not yet linking numerals on clock faces to units of time.	Tells time to the hour, knows the days of the week, months of the year.	Tells time to the half hour, finds dates on a calendar.	Tells time to five-minute intervals, uses a calendar to predict dates.	Calculates times using hours and minutes.

EARLY CHILDHOOD INVENTORY MATHEMATICS

Counting Development Class Checklist

Students	A. Oral counts less than 10	Counts less than ten objects		B. Oral counts more than 10	Counts at least 10 objects	Makes small sets		C. Oral counts by ones to 30	Counts more than 20 objects		D. Counts on by ones from any 2 digit number	Counts back by ones from 10		E. Counts forwards by 10s, 5s, 2s	Counts back by ones from 20		F. Counts forwards by 3s, 4s	Counts back by 1s from any 2 digit number	Counts on by 5s, 10s from any 2 digit number

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Numeration Development Class Checklist

Students	A Knows some numbers 0-10		B Knows all numbers 0-10	Orders numbers 1-10		C Knows some 2 digit numbers	Tells the number one after		D Knows all 2 digit numbers	Orders 2 digit numbers	Models and explains place value for 2 digit numbers	Tells the number one before		E Knows and orders 3 digit numbers	Models & explains place value for 3 digit numbers	Models and explains 10 & 100 more	Models and explains 10 & 100 less		F Knows numbers up to 5 digits	Orders 4 and 5 digit numbers	Explains place value for 4 digit numbers

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Addition & Subtraction Development Class Checklist

Students	A Can't use counters to model addition		B Models and counts all from one to add	Models and counts all from one to subtract		C Counts on from one number when adding	Counts back, down to or up from for subtraction		D Adds/ subtracts 1,2	Adds/ subtracts 0	Doubles	Turn around strategy	Sums to 10	Adding 10		E Fact families	Using larger numbers	Near doubles	Make to 10	Subtracting 10	Adding 9

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Multiplication & Division Development Class Checklist

Students	A Can't make equal groups or share		B Can make equal groups for multiplying	Can make equal groups when sharing	Counts all from one for multiplying and sharing		C Needed to fully model	Skip counts to find total		D Skip counts to find total for multiplying	Skip counts to find total for dividing	Uses part model to keep count of the groups		E Tens strategy for multiplying	Doubling strategy for multiplying	Repeated addition strategy for multiplying	Turn around strategy for multiplying		F Tens strategy for dividing	Halving	Repeated addition or subtraction

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Geometry Development Class Checklist

Students	A Unable to recognize shapes	Unable to name shapes		B Recognizes shapes in conventional orientation	Names shapes in conventional orientation		C Notices number of sides for shapes	Notices number of corners for shapes	Visualizes simple orientation of shapes		D Identifies shapes in non conventional orientation	Recognizes need for closure of shapes	Recognizes straight lines in defining shapes	Visualizes rotations and reflections of simple shapes	Composes and decomposes simple shapes		E Recognizes elements of lines – opposite, equal, parallel, perpendicular	Recognizes angles and their affect on shapes	Understands family relationships of 4 sided shapes	Visualizes rotations and reflections of many shapes

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Length Development Class Checklist

Students	A Compares lengths perceptually		B Compares lengths of two or more objects	Aligns one end of the objects accurately		C Understands use of non standard units	Avoids gaps and overlaps between unit	Uses name of the unit	Measures using repeated unit and markings		D Measures length using one system of formal units	Uses benchmarks to estimate length in one system	Measures accurately lengths less than a ruler length		E Measures accurately using Metric and Standard units	Measures lengths more than the length of the ruler	Uses benchmarks to estimate in both systems

EARLY CHILDHOOD INVENTORY IN MATHEMATICS

Time Development Class Checklist

Students	A Recognizes clocks refer to time	May know some days or months		B Tells time to the hour in analogue and digital	Knows days of the week and months of the year		C Tells time to the half hour in analogue and digital	Reads calendar:	- date, weekday, day before, day after	Knows some benchmarks - 3 o'clock home time		D Tells time to five minute intervals	Uses a calendar to find various dates and days	Makes forward predictions of time	Estimates time usage for minutes and hours		E Tells time accurately with both analogue and digital	Makes forward and backward predictions of time	Has strategies for estimating duration of time.	Estimates how long familiar activities might take

Early Childhood Inventory in Mathematics: Individual Student Record

Student _____

COUNTING

A	Oral counts less than 10	Counts less than 10 objects	
B	Oral counts more than 10	Counts at least 10 objects	Makes small sets
C	Oral counts by ones to 30	Counts more than 20 objects	
D	Counts back by ones from 10	Counts on by ones from any 2 digit number	
E	Counts forwards by 10s, 5s, 2s	Counts back by ones from 20	
F	Counts forwards by 3s, 4s	Counts on by 5s, 10s from any 2 digit number	Counts back by 1s

NUMERATION

A	Knows some numbers 0-10		
B	Knows all numbers 0-10	Orders numbers 1-10	
C	Knows some 2 digit numbers	Tells the number one after	
D	Knows & orders 2 digit numbers	Tells the number one before	Models and explains tens and ones
E	Knows, orders 3 digit numbers	Models & explains hundreds, tens and ones	Models 10 & 100 more, 10 & 100 less
F	Knows numbers to 5 digits Predicts 10 & 10 more, 10 & 100 less	Orders 4 & 5 digit numbers	Explains place value

ADDITION & SUBTRACTION

A	Can't yet model addition	Can't yet model subtraction	
B	Models and counts all from one to add	Models and counts all from one to subtract	Needs full model
C	Counts on to add	Counts back, down to, or up from to subtract	Needs part model
D	Adds, subtracts 1, 2 Turn around strategy	Adds, subtracts 0 Sums to 10	Doubles facts Adding 10
E	Fact families Make to 10	Using larger numbers Subtracting 10	Near doubles Adding 9

Early Childhood Inventory in Mathematics: Individual Student Record (cont.) **Student:**_____

MULTIPLICATION & DIVISION

A	Can't yet model equal groups	Can't yet share	
B	Makes and totals equal groups	Shares materials equally	Counts all from one
C	Skip counts or adds to total equal groups	Shares materials fairly	Divides materials into equal sized groups
D	Skip counts or adds to total equal groups	Skip counts, adds or subtracts to find quotient	Uses part model to keep count of groups
E	Tens strategy Turn around strategy	Doubling strategy	Repeated addition strategy
F	Tens strategy	Halving strategy	Repeated addition or subtraction strategy

GEOMETRY

A	Unable to recognize shapes	Unable to name shapes	
B	Recognizes shapes in conventional orientation	Names shapes in conventional orientation	
C	Notices numbers of sides	Notices numbers of corners	Visualizes simple orientation of shapes
D	Identifies shapes in non-conventional orientation Visualizes rotations of simple shapes	Recognizes need for closure of shapes Composes and decomposes simple shapes	Recognizes straight lines to define polygons
E	Recognizes elements of lines: equal length, opposite, parallel, perpendicular Visualizes rotations and reflections of many shapes	Recognizes angles and their effect on shapes Composes and decomposes shapes	Understands relationships between 4 sided shapes

LENGTH MEASUREMENT

A	Finds "longest" object but cannot describe length of other objects	Compares length perceptually	
B	Compares length of two or more objects	Describes length of two or more objects	Aligns one end of the objects accurately
C	Aligns non-standard units beside object to be measured Measures using repeated unit and markings	Avoids gaps and overlaps between units	Gives the number and the name of the unit
D	Measures length using one system of formal units	Uses benchmarks to estimate length in one system	Measures accurately lengths less than a ruler length
E	Measures accurately using Metric and Standard units	Measures lengths more than the length of a ruler	Uses benchmarks to estimate in both systems

Early Childhood Inventory in Mathematics: Individual Student Record (cont.) **Student:**_____

TIME MEASUREMENT

A	Recognizes clocks refer to time	May know some days or months	
B	Tells time to the hour Knows morning, afternoon, night	Knows days of the week	Knows months of the year
C	Tells time to the half hour	Knows some benchmark times	Reads calendar: day, date
D	Tells time to five minute intervals Uses calendar to find various dates	Calculates time after a given time	Explains activities that take minutes, hours
E	Tells time accurately Estimates time for a range of activities	Calculates time before a given time	Has strategies for estimating seconds, minutes

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